University of Nottingham
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Gears 3

Gear design analysis

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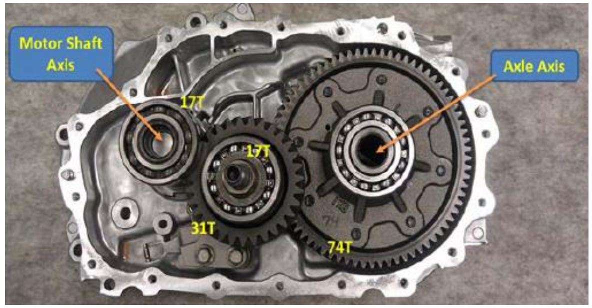
Gears related Lecture sessions

Gears 1
- **Introduction to gears**
  - Functions & types
  - Gear terminologies & conjugate action
  - Involute profile, fundamental equations, tooth system

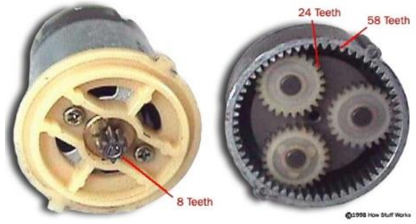
Gears 2
- **Gear trains and their applications**
  - Simple and compound trains
  - Planetary train
  - Differential unit
  - Applications

Gears 3
- **Gear stress analysis & design**
  - **Common forms of gear failure**
  - **Gear force analysis**
  - **AGMA gear stress analysis and design**

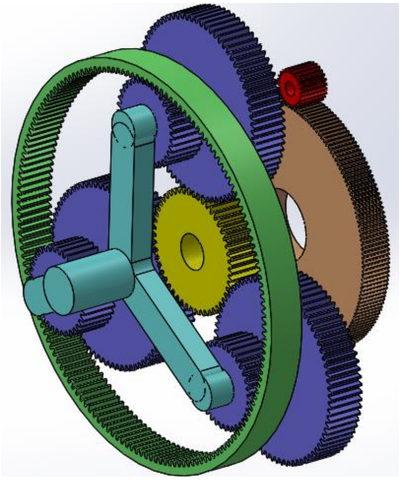
Future geared turbofan engine, like RR Ultrafan



Nissan Leaf gearbox



Planetary gearbox of mini hand drill



A SW model of WT gearbox

Outline of Gears 3

- **Gear stress analysis and design**

Part 1:
- Common forms of gear failure
- Gear force analysis

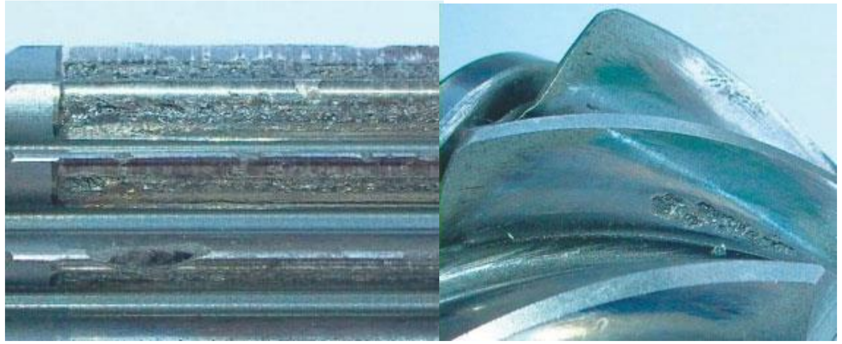
Part 2:
- Basic equations of gear **bending and contact stresses**

Part 3:
- AGMA based gear stress analysis and design
  - **AGMA bending & contact stress** calculations
  - **AGMA allowable bending & contact stresses** of a chosen material
  - General gear design procedure

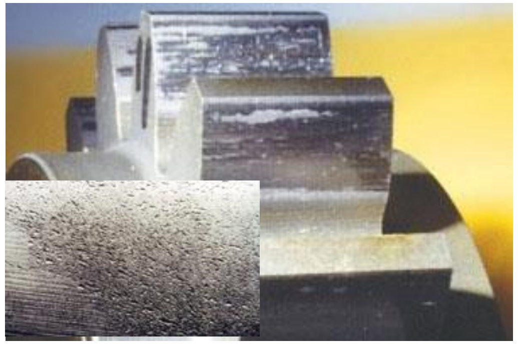
Part 4:
- A worked example

Common forms of gear failure

Bending fatigue (due to cyclic bending stress at tooth root)

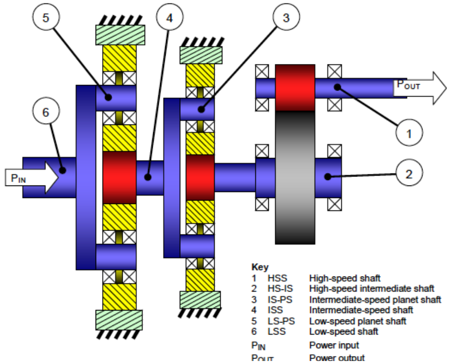


Pitting (common gear failure caused **by surface damage from cyclic contact stress**)
http://machinedesign.com/article/recognizing-gear-failures-0621



Micropitting (surface failure due to use of surface hardened gears, small craters)

Scuffing (adhesive wear which instantly damages tooth surfaces in relative motion)



A **1.5MW WT**, Portsmouth, RI, **Gearbox failed aft 3yrs** operation with a **20yrs design life**.

Gearbox configuration: 2 planetary stages + 1 stage of parallel gears with a ratio of 1:120 to convert **17rpm** from rotor to **2000rpm** of generator

https://www.wind-watch.org/documents/gearbox-failure-investigation/

1st stage planetary ring

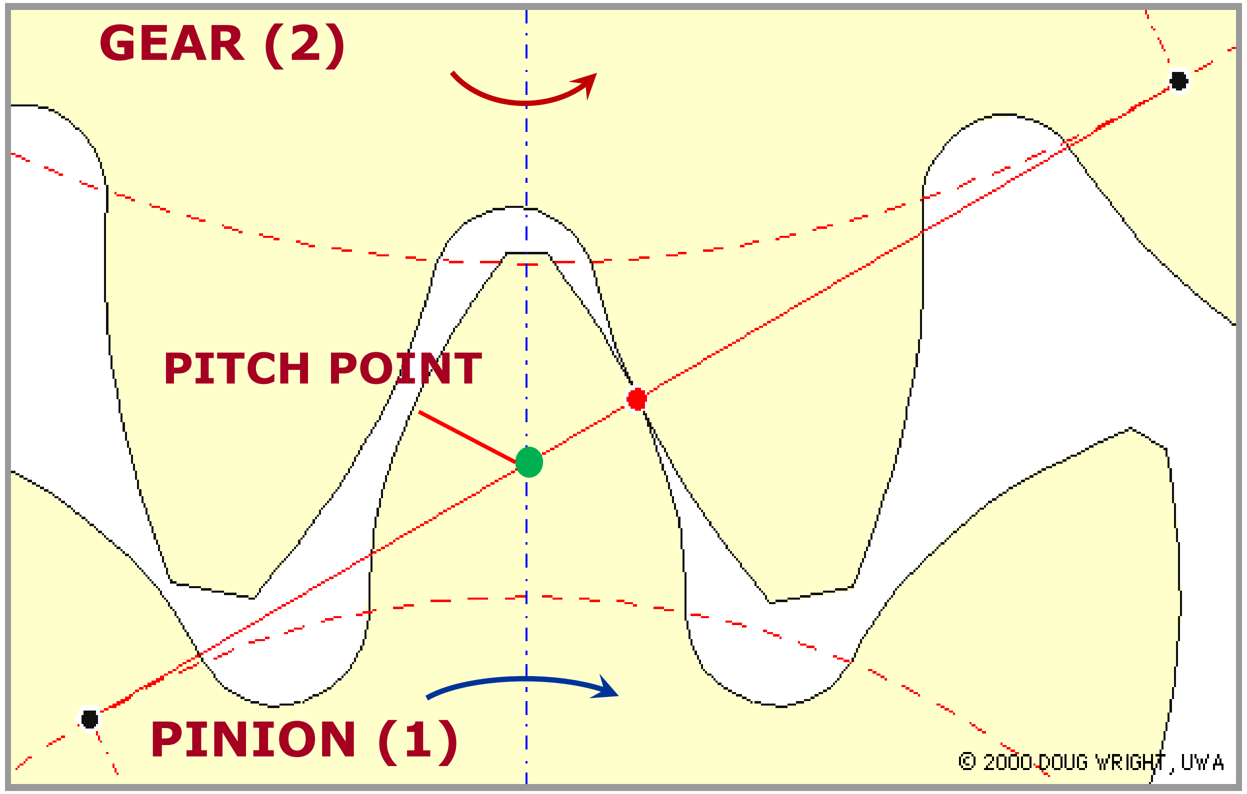
1st stage planetary ring

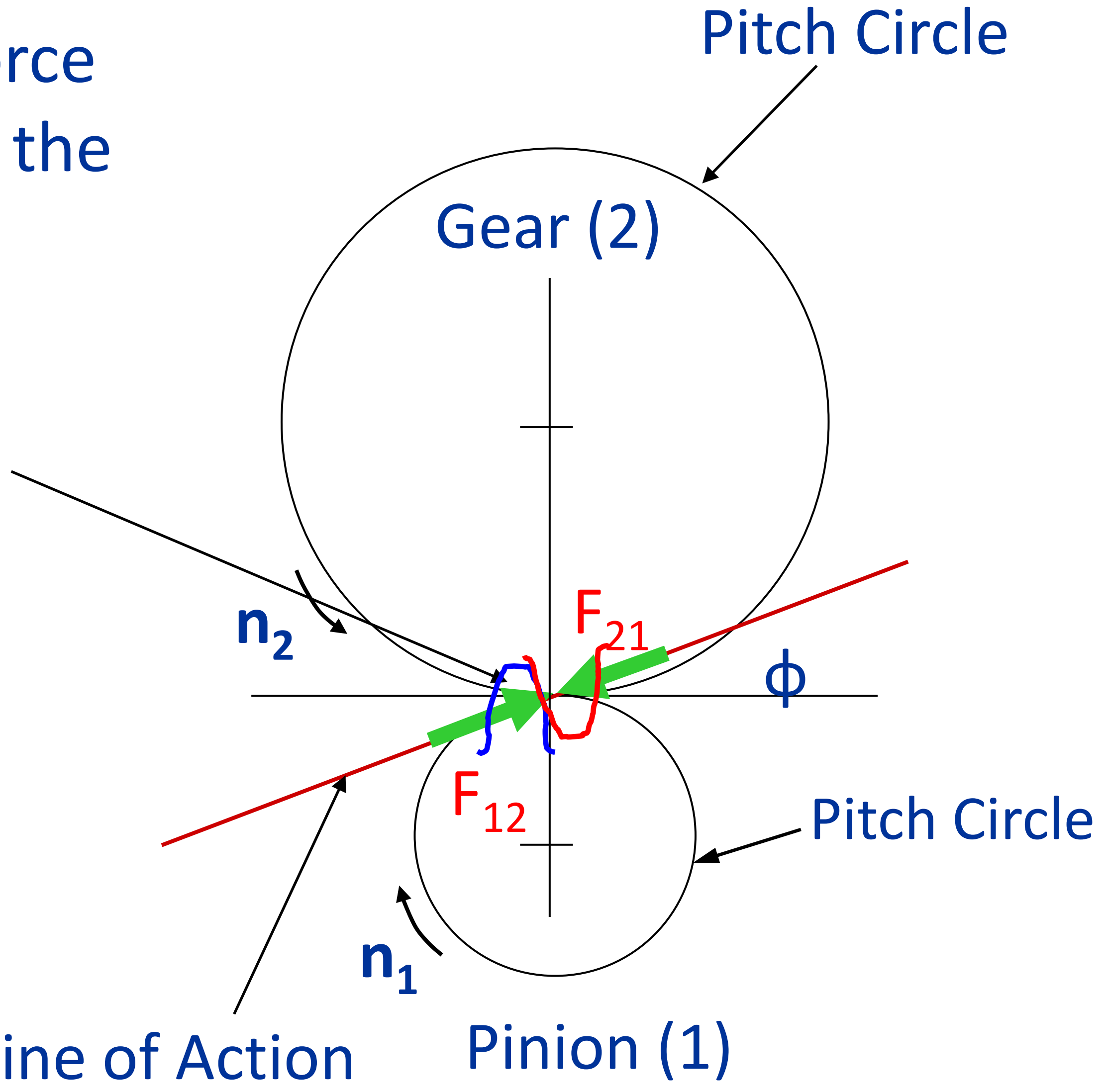
Key questions for gear design analysis

- What is the mechanism of forces transmitted and how to quantify them in gear meshing operations?
- What are the typical forms of stresses and how can they be accurately calculated?
  - ✓ Methods of determining stresses under ideal conditions
  - ✓ Considerations given to accommodate the effects of real working conditions
- With selected material and manufacturing methods, how to quantify the strength or allowable stress of a gear train?

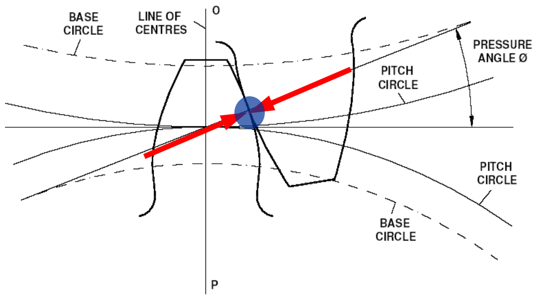
Gear force analysis

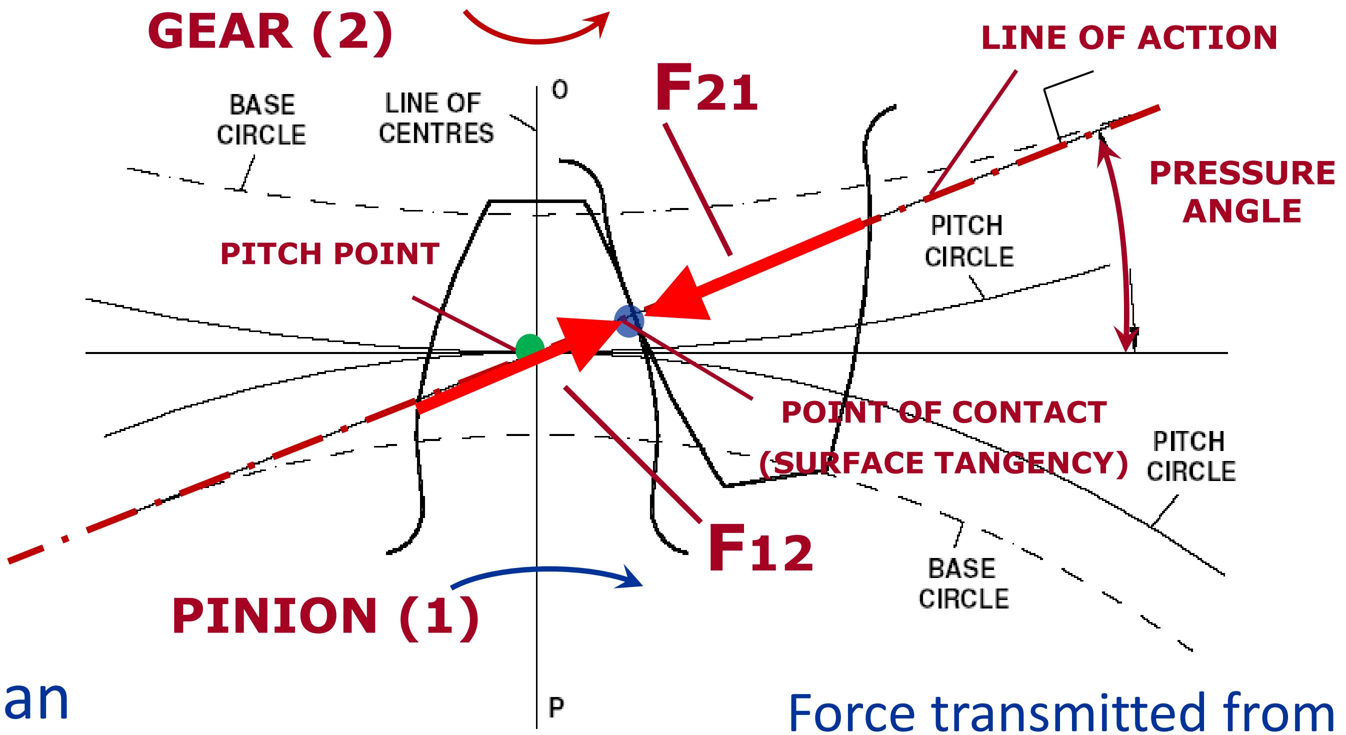
Gear Design Calculation is based on force transmitted along the line of action at the **Pitch Point**





Gear force analysis





For two spur gears in mesh at an arbitrary point on the **line of action**

Force transmitted from the pinion to the gear

F12 - Force by the Pinion (1) on Gear (2)

F21 – Force by the Gear (2) on Pinion (1)

Gear force analysis

F_{21}^{T} = Force by Gear 2 on Pinion 1 - Tangential

W_T = Transmitted load, i.e. $W_T = F_{21}^{T}$

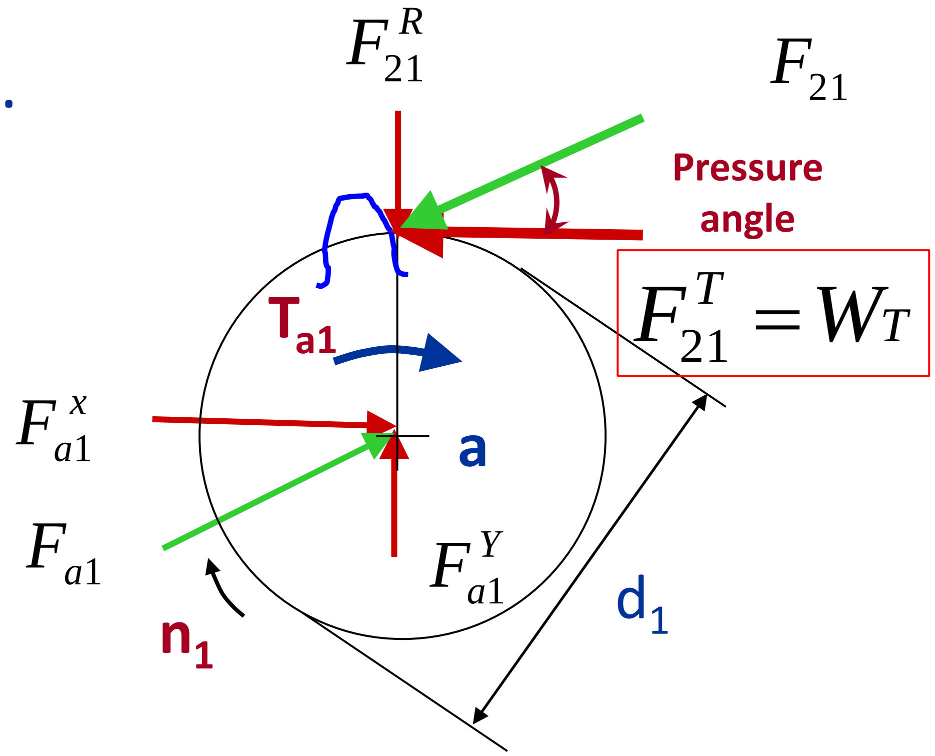
T_{a1} = Torque exerted by Shaft **a** on Pinion (1), i.e.

$$T_{a1} = \frac{W_T d_1}{2} \qquad P = T_{a1}\omega_1 \quad \text{or} \quad P = W_T V_{\frac{d_1}{2}}$$

$$V_{\frac{d_1}{2}} = \frac{d_1}{2}\omega_1, \qquad \omega = \frac{2\pi}{60} n(rpm)$$

where, ***P*** is power (kW), ***d1*** is pitch diameter of the pinion (mm), ***ω1*** is pinion speed (rad/s) and ***n1*** is pinion speed (rpm).

$$W_T = \frac{60\times 10^3 P}{\pi d_1 n_1}(kN) \quad \text{or} \quad W_T = \frac{P}{V_{\frac{d_1}{2}}}$$



Free Body Diagram: Forces acting on Pinion (1)

Gear force analysis

F_{12}^{T} = Force by Pinion (1) on Gear (2) - Tangential $W_T = F_{12}^{T}$

F_{12}^{R} = Force by Pinion (1) on Gear (2) - Radial

F_{b2}^{x} = Force by Shaft **b** on Gear (2)–x Direction

F_{b2}^{y} = Force by Shaft **b** on Gear (2)-y Direction

T_{b2} = Torque exerted by Shaft **b** on Gear (2)

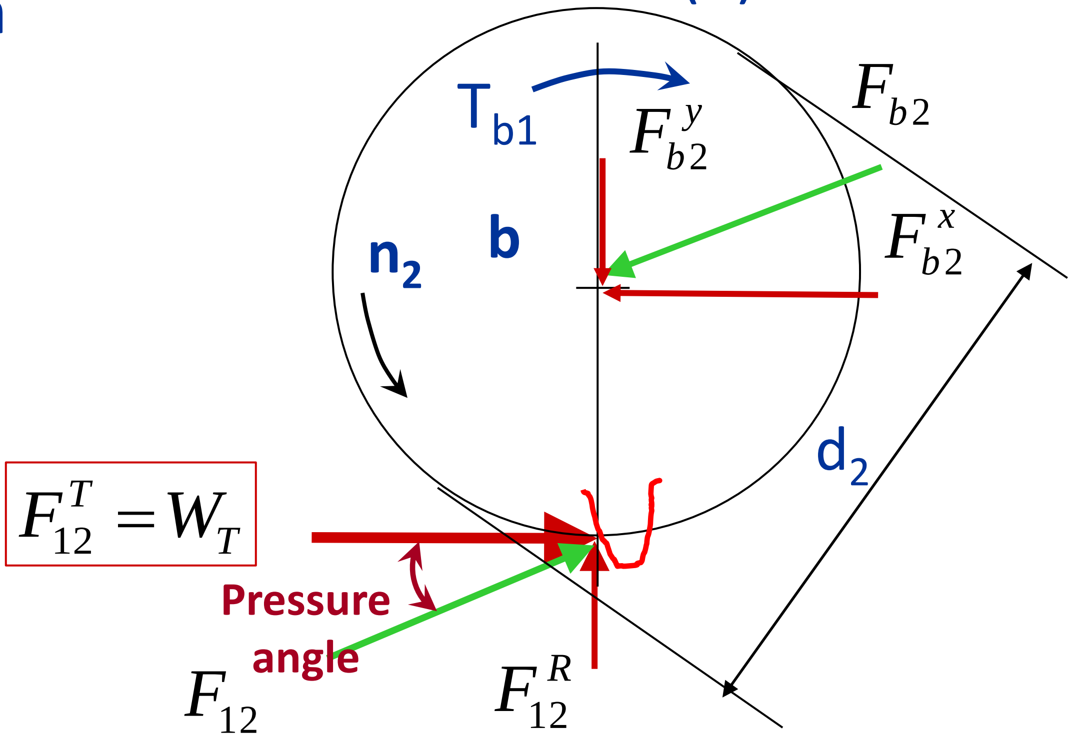
$$T_{b2} = \frac{W_T d_2}{2}$$

Transmitted load on Gear (2)

$$W_T = \frac{60 \times 10^3 P}{\pi d_1 n_1} (kN)$$

or

$$W_T = \frac{P}{V_{\frac{d_1}{2}}}$$



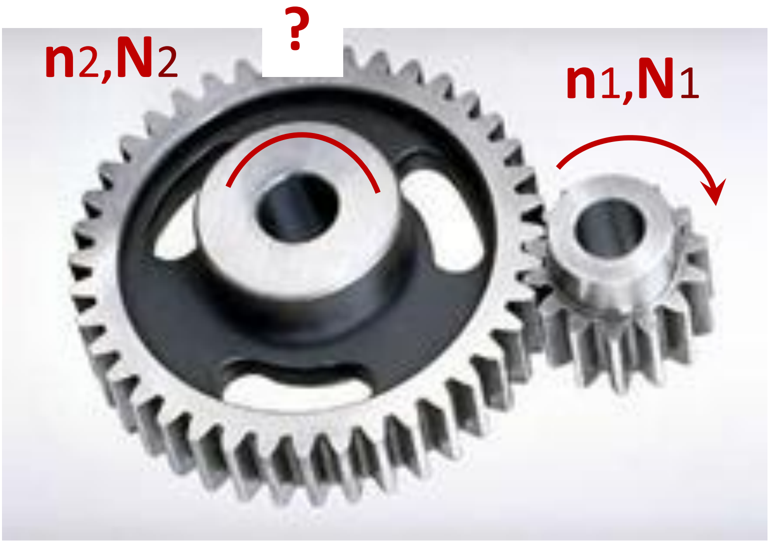
Free Body Diagram: Forces acting on Gear (2)

Worked example 1: transmitted load

For a pair of spur gears, the **module** for the pinion and gear is $m=2$ mm. The **numbers of teeth** for the pinion and gear, are $N_1=20$ and $N_2=40$, respectively . The rotating speed of the pinion is $\omega_1=900$ rpm (clockwise). The rated power for this gear set is $P=0.94$ kW.

Determine:

1) the speed and direction of rotation of the gear n_2,
2) the transmitted load W_T of the gear set.



Worked example 1: solution

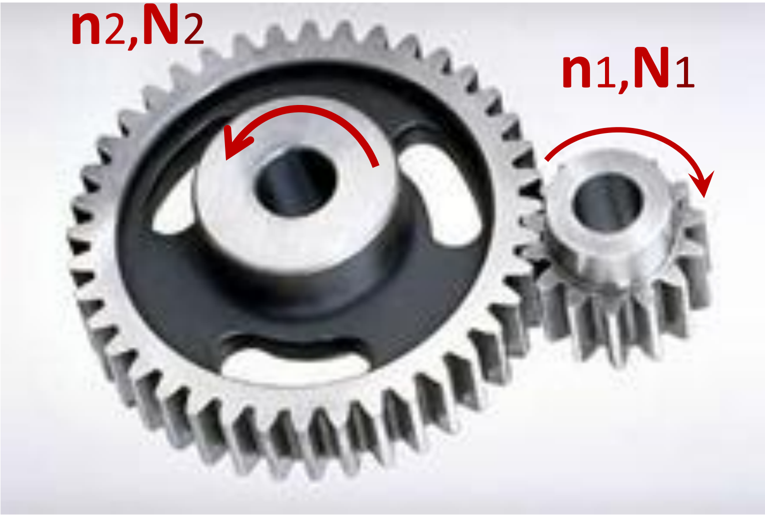
1) The speed and direction of the gear, ***ω2*:**

Use the gear ratio equation:

$$Z = \frac{\omega_1}{\omega_2} = -\frac{N_2}{N_1}$$

$$n_2 = -\frac{N_1}{N_2} n_1 = -\frac{20}{40} \times 900 = -450\ (rpm)$$

The direction of rotation of the gear is **anti-clockwise.**



2) The transmitted load, ***WT:***

The pitch diameter of the pinion,

$$\boldsymbol{d_1} = \boldsymbol{mN_1} = 2 \times 20 = 40(mm)$$

The transmitted load, ***WT***

$$V_{\frac{d_1}{2}} = \frac{d_1}{2}\frac{2\pi}{60} n_1 = 1.85\ (^{m}/_{s})$$

$$\boldsymbol{W_T} = \frac{\boldsymbol{60 \times 10^3\ P}}{\boldsymbol{\pi d_1 n_1}} = \frac{60 \times 10^3 \times 0.94}{3.1416 \times 40 \times 900} = 0.5(kN)$$

$$\boldsymbol{W_T} = \frac{\boldsymbol{10^3 P}}{\boldsymbol{V_{\frac{d_1}{2}}}} = \frac{10^3 \times 0.93}{1.85} = 502.7(N)$$

Gears 3

End of Part 1

Gears 3

Part 2

Key questions for gear design analysis

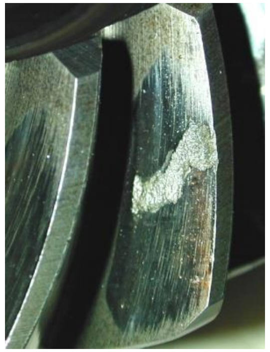
- What is the mechanism of forces transmitted and how to quantify them in gear meshing operations?

- What are the typical forms of stresses and how can they be accurately calculated?
  - ✓ Methods of determining stresses under ideal conditions
  - ✓ Considerations given to accommodate the effects of real working conditions

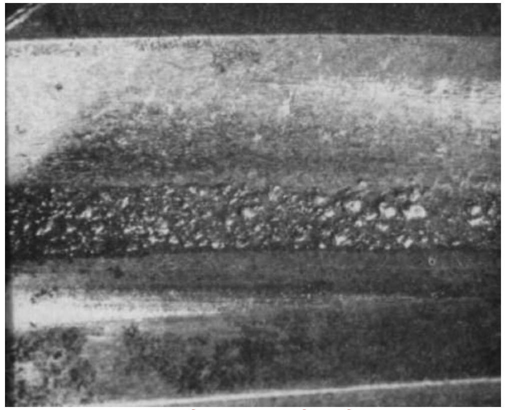
- With selected material and manufacturing methods, how to quantify the strength or allowable stress of a gear train?

Gear stress analysis

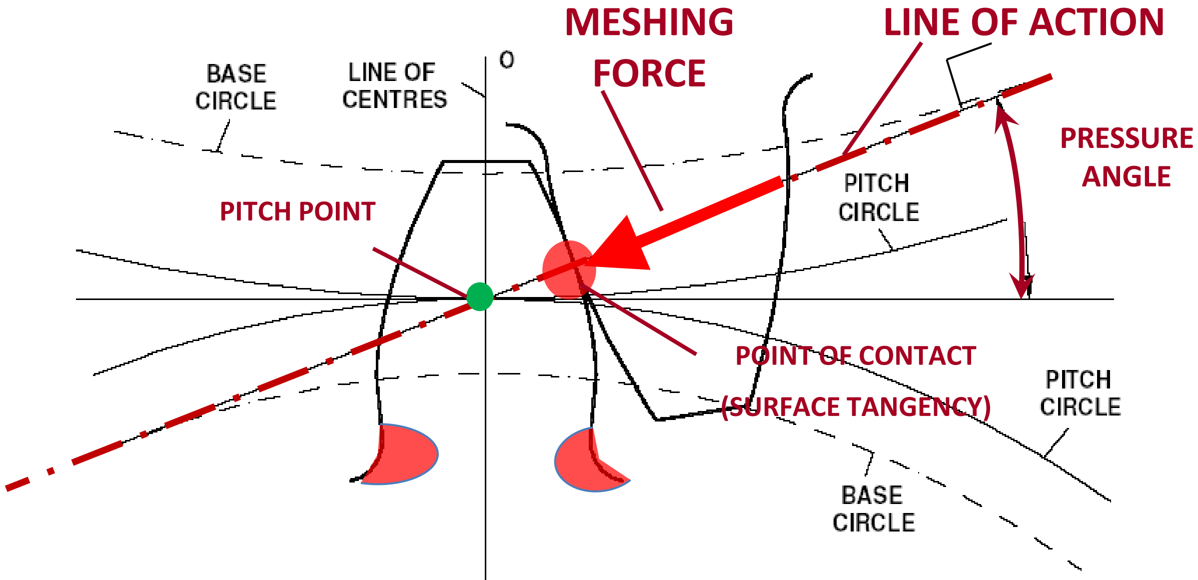
Tooth bending fatigue



Pitting (surface fatigue failure)



Micro pitting



Force transmitted from the pinion to the gear

- Force is always normal to the meshing point along the **line of action**,
- which generates a **bending moment thus stress** at the root of the gear &
- a stress concentration called **contact stress** at the meshing point

Spur gear stress analysis

Gears experience **two types of stresses**:

1. **Bending stress** (at the root of tooth)
2. **Contact stress** (on teeth faces) due to meshing

Standards for Gear Stress Analysis:

- **AGMA** (American Gear Manufacturers Association) – **(ANSI/AGMA 2101-C95)** (~70 pages)
- **BS ISO 6336-1~6: 2006** (~300 pages)

Approach:

1. Calculate **maximum bending and contact stresses** in gears
2. Compare to **allowable bending and contact stresses** with a chosen gear material

In using **AGMA or BS/ISO** in gear design calculation, **it is important to**

- understand basic concepts & methods, important assumptions;
- be careful of many parameters (units) & empirical nature of affecting factors.

Basic equation for gear bending stress

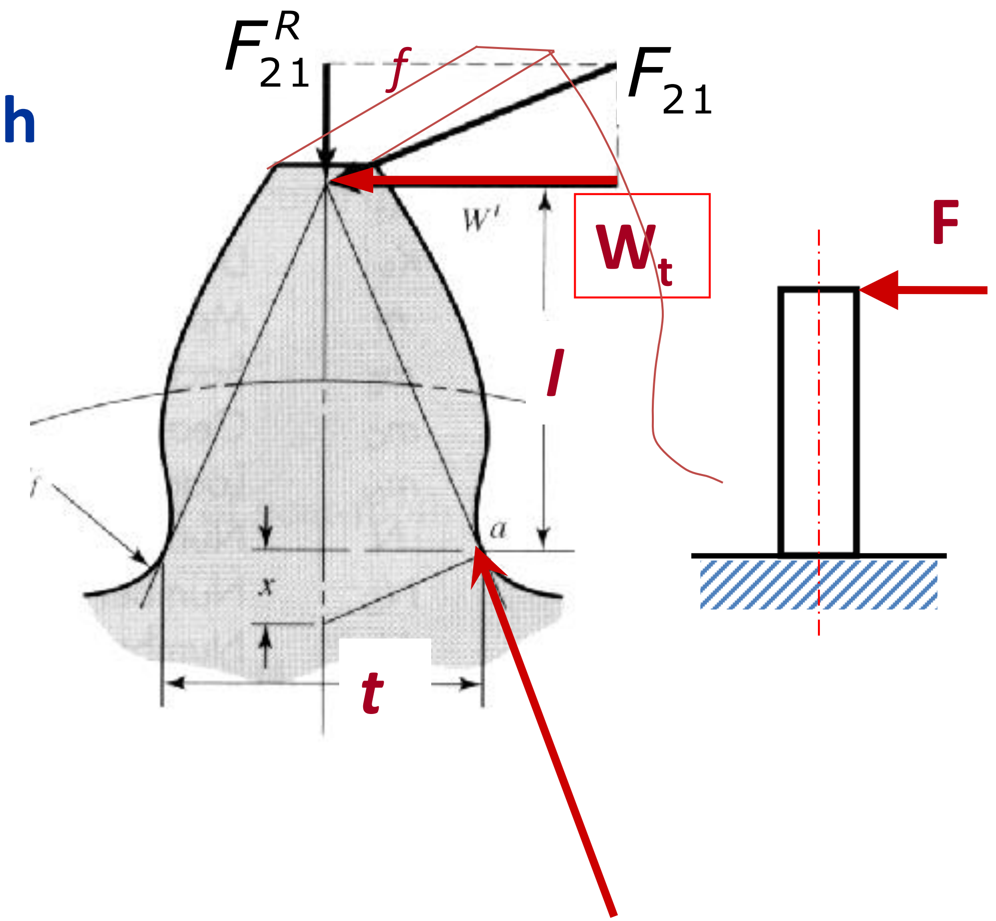
Gear tooth may be simplified as a cantilever beam (**Lewis Bending Equation, 1892**)

Worst case assumption:

- **W_t (transmitted Load) applied at the top of tooth**
- **One pair of teeth in contact.**

Maximum Bending Stress:

$$\sigma_b = \frac{M \cdot y}{I} = \frac{(W_t \cdot l) \cdot \left(\frac{t}{2}\right)}{\left(\frac{f \cdot t^3}{12}\right)} = \frac{6 \cdot W_t \cdot l}{f \cdot t^2}$$



σ_b=Maximum Bending Stress

W_t = transmitted load

I = *2nd moment of area* ($I = \frac{f \cdot t^3}{12}$)

l = height of tooth

t = thickness of tooth at flank

f = face width of tooth

Basic equation for gear bending stress

Rearrangement of the maximum bending stress

σ_b=Maximum Bending Stress

$$\sigma_b = \frac{W_t}{F\,mY}$$

W_t = transmitted load

F = face width

m = Module

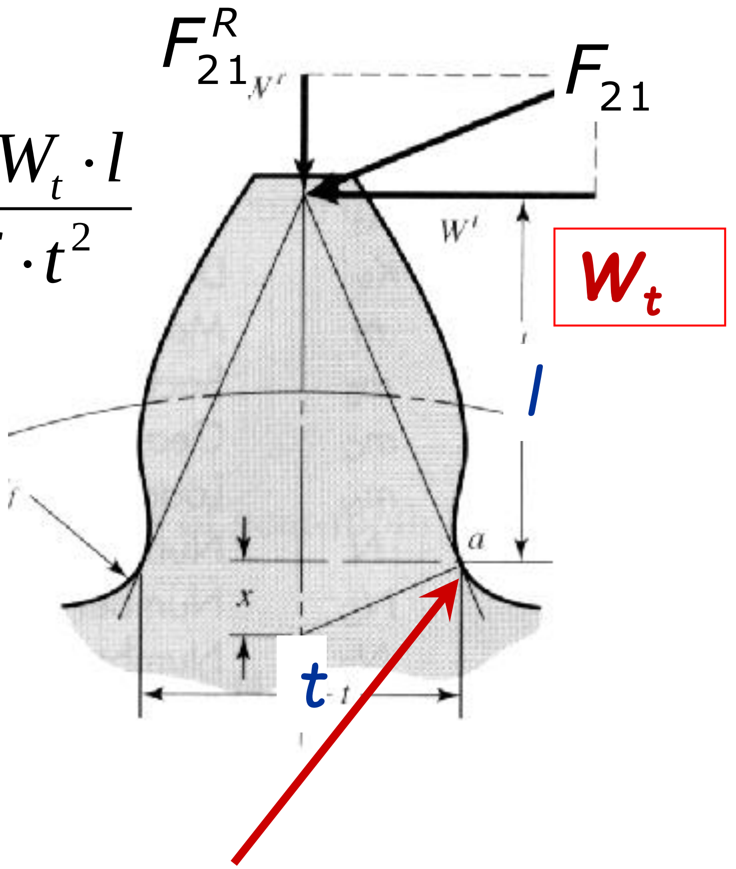
Y = geometry factor

$$\sigma_b = \frac{6 \cdot W_t \cdot l}{f \cdot t^2}$$

$$\frac{t/2}{x} = \frac{l}{t/2} \rightarrow t^2 = 4xl$$

and assume

$$Y = \frac{2}{3}\,x/m$$



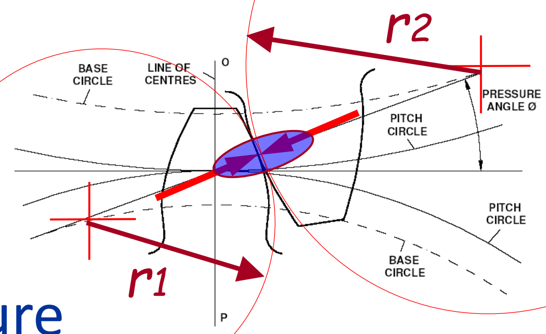
σ_b=Maximum Bending Stress

Geometry factor *Y* accounts for stress concentration at root of tooth

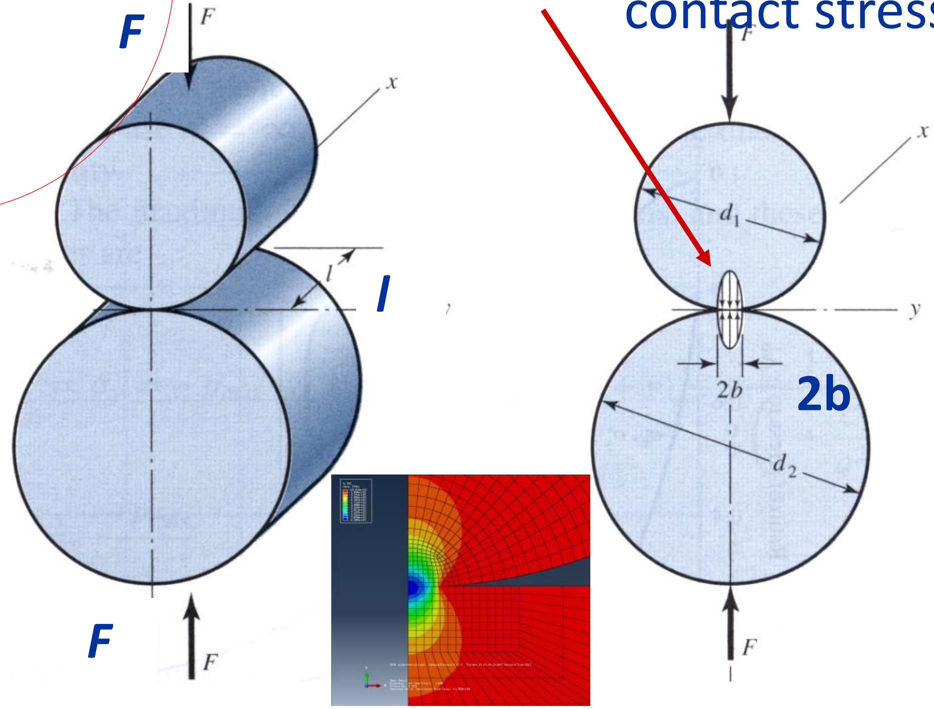
Basic equation for gear contact stress

Contact stress between two cylinders (Hertzian contact stress equation, 1882)

$$p_{max} = \frac{2F}{\pi b l}$$



pmax (σ_c) = maximum contact stress



pmax = maximum surface pressure

F = force applied to the cylinders

l = length of cylinders

b = width of the contact zone, and

$$b = \left\{ \frac{2F}{\pi l} \frac{\left[\left(1-\nu_1^2\right)/E_1\right] + \left[\left(1-\nu_2^2\right)/E_2\right]}{\left(1/d_1\right) + \left(1/d_2\right)} \right\}^{1/2}$$

so

$$p_{max} = \sqrt{\frac{F}{\pi l} \frac{\left(1/r_1 + 1/r_2\right)}{\left[\left(1-\nu_1^2\right)/E_1 + \left(1-\nu_2^2\right)/E_2\right]}}$$

$1/r = \left(1/r_1 + 1/r_2\right)$ effective radius between two cylinders

E1, E2, are Young's Modulus and

ν1, ν2 are Poisson's ratio

Basic equation for gear contact stress

Replacing ***F*** by *Wt/cosϕ* and ***l*** by the **Face width *F***

$$\sigma_C = \left\{ \frac{W_t}{\pi F \cos\phi} \frac{[(1/r_1)+(1/r_2)]}{[(1-\nu_1^2)/E_1]+[(1-\nu_2^2)/E_2]} \right\}^{1/2}$$

$$p_{\max} = \sqrt{\frac{F}{\pi l} \frac{(1/r_1 + 1/r_2)}{[(1-\nu_1^2)/E_1 + (1-\nu_2^2)/E_2]}}$$

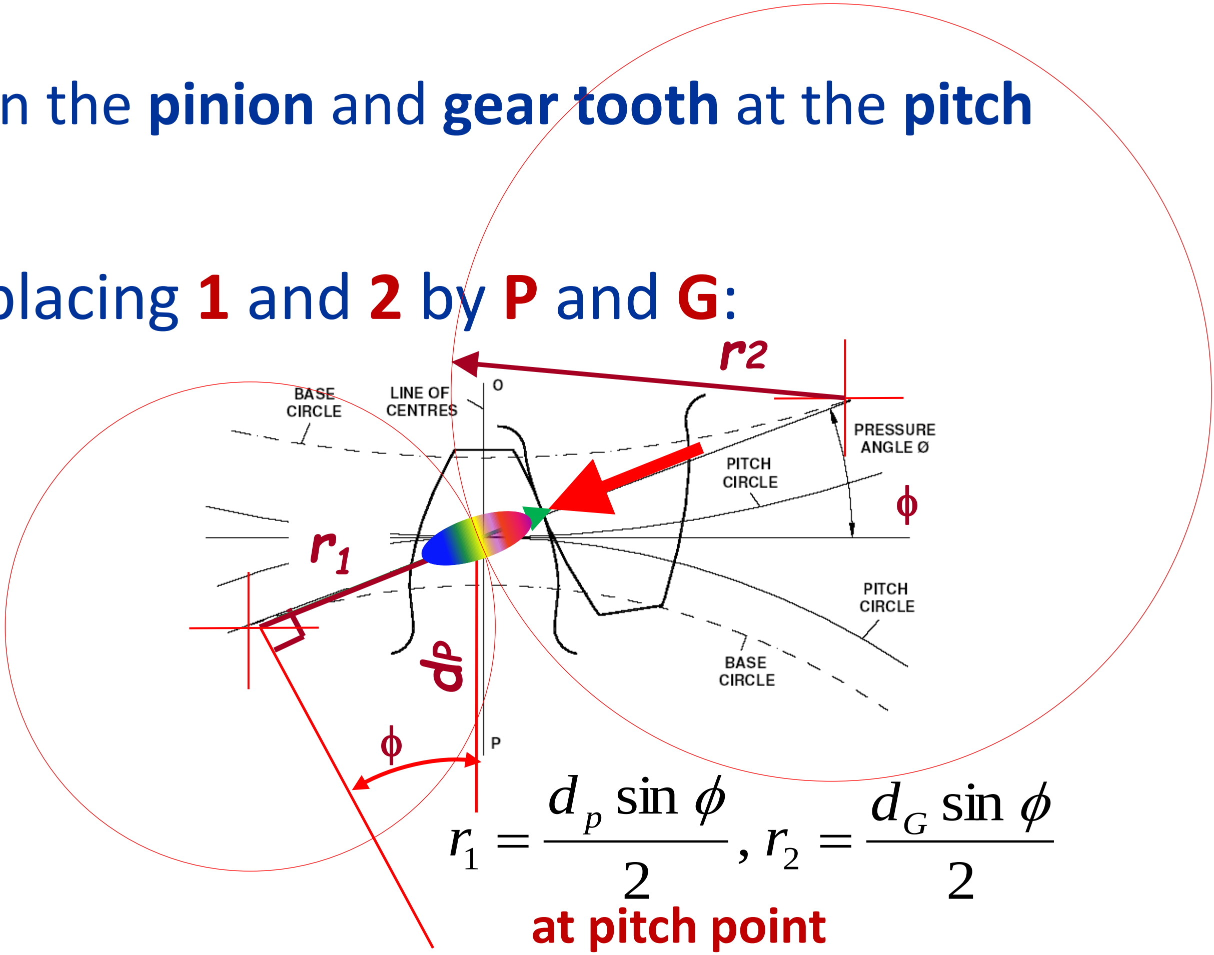
r_1*, *r_2 are the instantaneous radii of curvature on the **pinion** and **gear tooth** at the **pitch point of contact**.

Introducing an elastic coefficient, ***Ze*** and replacing **1** and **2** by **P** and **G**:

$$Z_e = \left\{ 1 \Big/ \left[\pi \left(\frac{1-\nu_P^2}{E_P} + \frac{1-\nu_G^2}{E_G} \right) \right] \right\}^{1/2}$$

Therefore, **maximum contact stress**

$$\sigma_C = Z_e \left[\frac{W_t}{F \cos\phi} \left(\frac{1}{r_1} + \frac{1}{r_2} \right) \right]^{1/2}$$



$$r_1 = \frac{d_p \sin\phi}{2}, \quad r_2 = \frac{d_G \sin\phi}{2}$$

at pitch point

Gears 3

End of Part 2

Gears 3

Part 3

Key questions for gear design analysis

- What is the mechanism of forces transmitted and how to quantify them in gear meshing operations?

- What are the typical forms of stresses and how can they be accurately calculated?
  - ✓ Methods of determining stresses under ideal conditions
  - ✓ Considerations given to accommodate the effects of real working conditions **based on AGMA standard**

- With selected material and manufacturing methods, how to quantify the strength or allowable stress of a gear train?

AGMA equations for **bending** stress (ANSI/AGMA 2101-C95)

Gear bending stress

$$\sigma = W_t K_O K_V^{'} K_S \frac{1}{F\,m} \frac{K_H K_B}{Y_J}$$

σ = Max bending stress (N/mm²)

W_t = Transmitted load (N)

F = Face width (mm)

m = Module (mm)

Y_J = geometry factor including stress concentration

K_O = overload factor

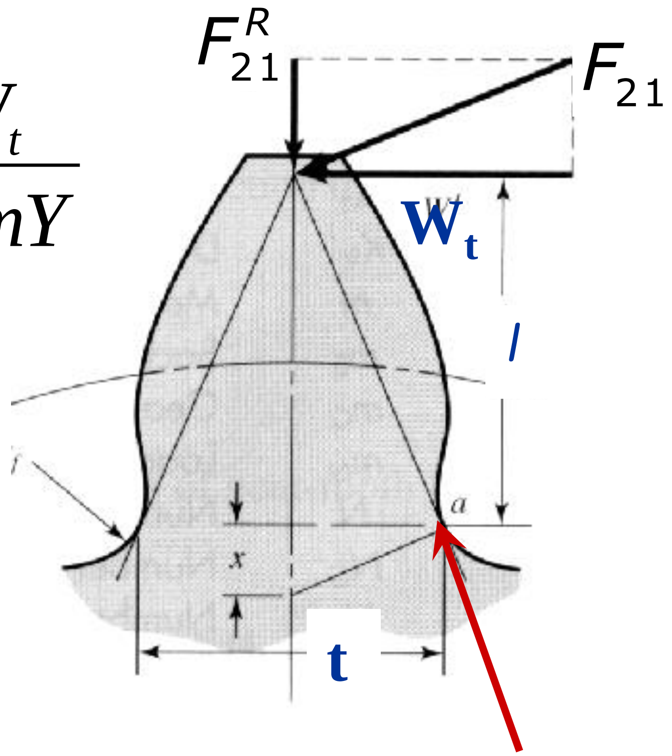
K'_V = dynamic factor

K_S = size factor

K_H = load-distribution factor

K_B = rim-thickness factor

$$\sigma = \frac{W_t}{F\,mY}$$



Maximum Bending Stress

AGMA equations for **bending** stress

$$\sigma = W_t K_O K_V' K_S \frac{1}{Fm} \frac{K_H K_B}{Y_J}$$

Ko = **overload factor**, to account for externally applied loads in excess of the nominal transmitted load $\boldsymbol{W_t}$, ***Ko= 1 ~ 2.75***.

K'v = **dynamic factor**, to account for inaccuracies in manufacturing and meshing of gear teeth at different speeds.

$$K_V' = \left(\frac{A + \sqrt{200V}}{A} \right)^B$$ where, $A = 50 + 56(1 - B)$ and $B = \frac{(12 - Q_V)^{2/3}}{4}$

V = velocity of gear (m/s);

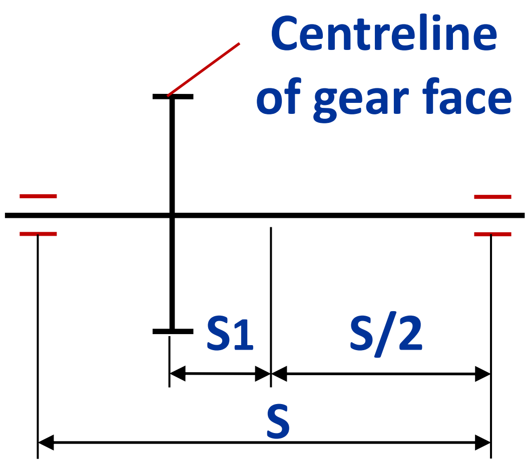
Qv = AGMA **transmission accuracy** level number,

3 ≤ *Qv* ≤ 7 for most commercial quality gears, agricultural & plant machinery, etc.

8 ≤ *Qv* ≤ 12 for precision quality gears, power tools & cars.

AGMA equations for **bending** stress

$$\sigma = W_t K_O K_V' K_S \frac{1}{Fm}\frac{K_H K_B}{Y_J}$$



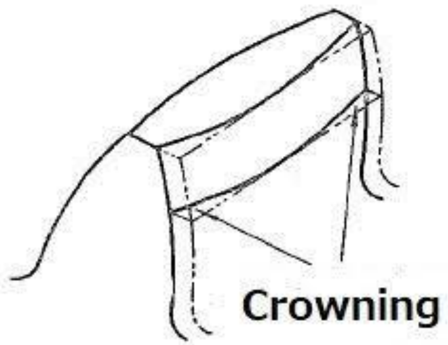
Ks = **size factor**, to account for non-uniformity of material properties due to size, normally ***Ks* = 1**.

KH = **load-distribution factor**, to account for non-uniform distribution of load across **tooth face width (*F*)**.

$$K_H = 1 + C_{mc}\left(C_{pf}C_{pm} + C_{ma}C_e\right)$$

$$\begin{cases} \text{if } F \le 0.0254(m), & C_{pf} = \dfrac{F}{10d} - 0.025 \\ \text{if } 0.0254(m) < F \le 0.4318(m), & C_{pf} = \dfrac{F}{10d} - 0.0375 + 0.4921F \\ \text{if } 0.4318(m) < F \le 1.016(m), & C_{pf} = \dfrac{F}{10d} + 0.815F - 0.3534F^2 \end{cases}$$

for F/10d <0.05, set F/10d=0.05, F=face width & d=pitch diameter



crowned tooth

$C_{mc} = \begin{cases} 1 \\ 0.8 \end{cases}$ **for uncrowned teeth** / **for crowned teeth**

$$C_{pm} = \begin{cases} 1, & \text{for } S_1/S < 0.175 \\ 1.1, & \text{for } S_1/S \ge 0.175 \end{cases}$$

(See above Figure)

$$C_{ma} = A + BF + CF^2$$

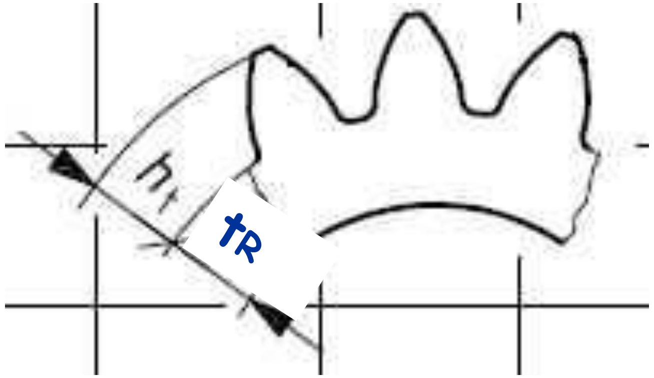
Condition	A	Bx10-3	Cx10-7
Commercial gear units	0.127	0.622	-1.69
Precision gear units	0.0675	0.504	-1.44

$C_e = \begin{cases} 0.8 \\ 1. \end{cases}$ **for gearing adjusted at assembly** / **for all other conditions**

AGMA equations for **bending** stress

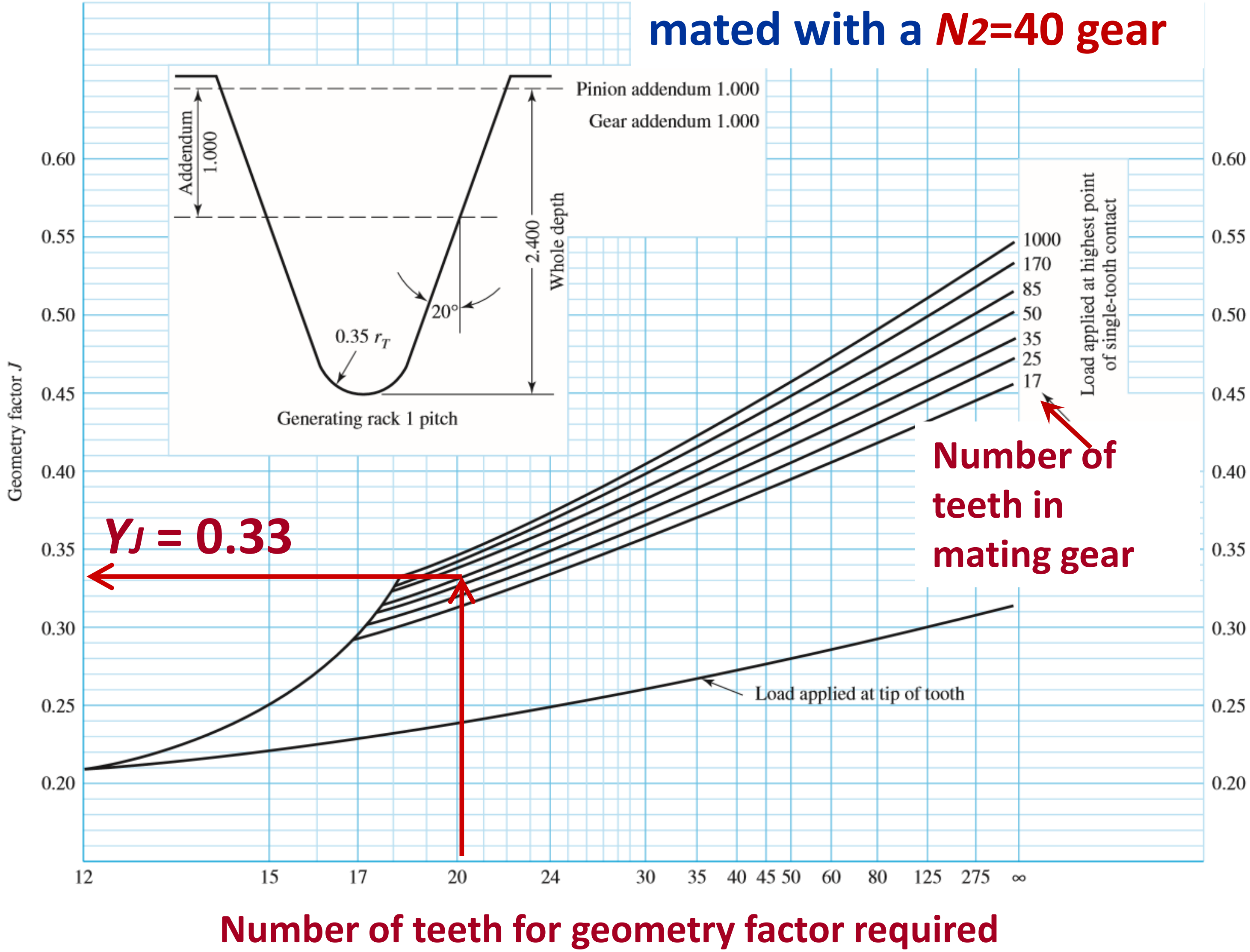
$$\sigma = W_t K_O K_V' K_S \frac{1}{Fm} \frac{K_H K_B}{Y_J}$$

K_B = **rim-thickness factor**, to account for adjustment of the estimated bending stress for the thin-rimmed gear, ***K_B* = 1 when *t_R*/*h_t*≥1.2.**



Y_J = **geometry factor** for bending strength, a modified value for the Lewis form factor & stress concentration (also in **Appendix**).

For example, find *Y_J* for a *N1*=20 pinion, which is mated with a *N2*=40 gear



AGMA equations for **contact** stress (ANSI/AGMA 2101-C95)

Gear contact stress (pitting resistance)

$$\sigma_C = Z_e \sqrt{W_t K_O K_V^{'} K_S \frac{K_H}{F d_p} \frac{Z_R}{Z_I}}$$

$$\sigma_C = Z_e \left[\frac{W_t}{F \cos\phi} \left(\frac{1}{r_1} + \frac{1}{r_2} \right) \right]^{1/2}$$

σ_C = contact stress (MPa)

W_t = transmitted load (N)

F = face width (mm)

d_p = pitch diameter(mm)

Z_e = elastic coefficient ($MPa^{0.5}$)

K_O = overload factor

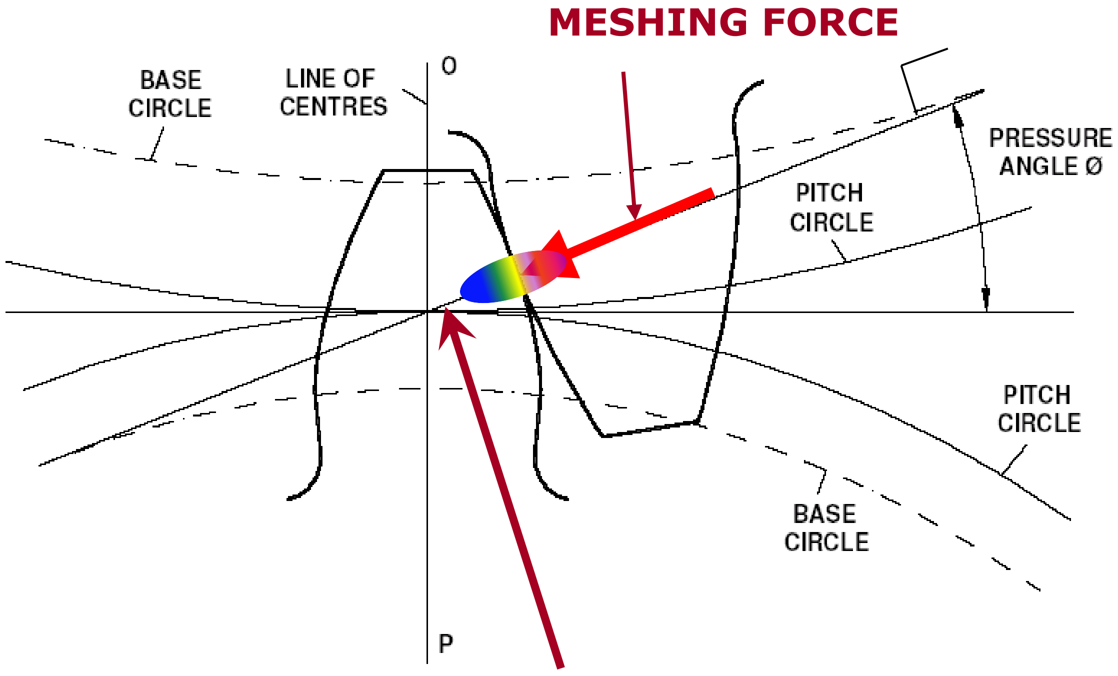
K'_V = dynamic factor

K_S = size factor

K_H = load-distribution factor

Z_R = surface condition factor

Z_I = geometry factor



Maximum Contact Stress

AGMA equations for **contact** stress

$$\sigma_C = Z_e \sqrt{W_t K_O K_V' K_S \frac{K_H}{F d_p} \frac{Z_R}{Z_I}}$$

$$\sigma_C = \sqrt{\frac{W_t}{\pi F \cos\phi} \frac{[(1/r_1)+(1/r_2)]}{[(1-\nu_1^2)/E_1]+[(1-\nu_2^2)/E_2]}}$$

Ze = an **elastic coefficient** (MPa$^{0.5}$), directly from Hertzian equation

$$\sigma_C = Z_e \left[\frac{W_t}{F \cos\phi} \left(\frac{1}{r_1} + \frac{1}{r_2} \right) \right]^{1/2}$$

$$Z_e = \left\{ 1 \Big/ \left[\pi \left(\frac{1-\nu_P^2}{E_P} + \frac{1-\nu_G^2}{E_G} \right) \right] \right\}^{1/2}$$

ZR = **surface condition factor**, to account for surface finish, residual stress & work hardening.

(Standard surface conditions **have not yet been established**. When a detrimental surface finish effect is known to exist, **AGMA suggests a value of greater than 1, i.e. ZR≥1.**)

AGMA equations for **contact** stress

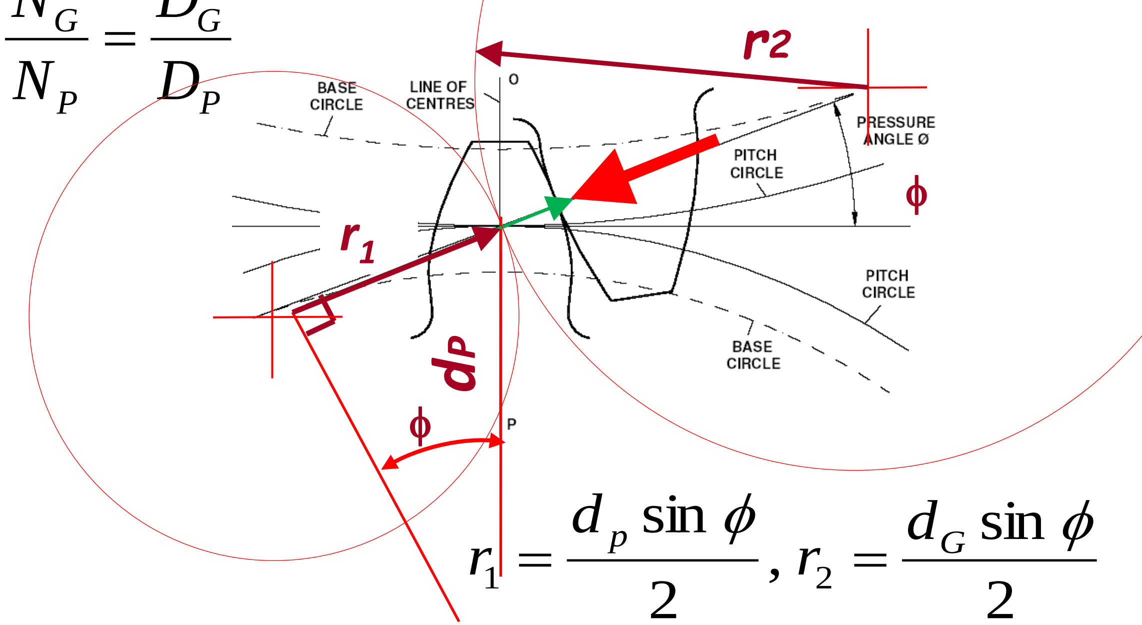
$$\sigma_C = Z_e \sqrt{W^t K_O K_V' K_S \frac{K_H}{F d_p} \frac{Z_R}{Z_I}}$$

$$\sigma_C = Z_e \left[\frac{W^t}{F \cos\phi} \left(\frac{1}{r_1} + \frac{1}{r_2} \right) \right]^{1/2}$$

***Z_I* = geometry factor** for contact strength,

As $r_1 = \dfrac{d_p \sin\phi}{2}$, $r_2 = \dfrac{d_G \sin\phi}{2}$ and $m_G (= Z) = \dfrac{N_G}{N_P} = \dfrac{D_G}{D_P}$

then $\dfrac{1}{r_1} + \dfrac{1}{r_2} = \dfrac{1}{d_p} \dfrac{2}{\sin\phi} \dfrac{m_G + 1}{m_G}$ at pitch point



$$r_1 = \frac{d_p \sin\phi}{2}, \quad r_2 = \frac{d_G \sin\phi}{2}$$

at pitch point.

$$\sigma_C = Z_e \left[\frac{W^t}{d_P F} \frac{1}{\dfrac{\cos\phi \sin\phi}{2} \dfrac{m_G}{m_G + 1}} \right]^{1/2}$$

Therefore,

$$Z_I = \frac{\cos\phi \sin\phi}{2} \frac{m_G}{m_G + 1}$$

Summary of AGMA gear **bending** and **contact** stress equations

Gears 3 Part 3

- **AGMA equations for bending & contact stresses**

Bending stress

$$\sigma = W_t K_O K_V^{'} K_S \frac{1}{F\,m} \frac{K_H K_B}{Y_J}$$

Contact stress

$$\sigma_C = Z_e \sqrt{W_t K_O K_V^{'} K_S \frac{K_H}{F d_p} \frac{Z_R}{Z_I}}$$

Key questions for gear design analysis

- What is the mechanism of forces transmitted and how to quantify them in gear meshing operations?

- What are the typical forms of stresses and how can they be accurately calculated?
  - ✓ Methods of determining stresses under ideal conditions
  - ✓ Considerations given to accommodate the effects of real working conditions **based on GAMA standard**

- With selected material and manufacturing methods, how to quantify the strength or allowable stress of a gear train?

AGMA **allowable bending stress** (ANSI/AGMA 2101-C95)

AGMA equation for the **allowable bending stress**

$$\sigma_{all} = \frac{\sigma_{FP}}{S_F} \frac{Y_N}{Y_\theta Y_Z}$$

σ_{FP} = allowable bending stress (MPa)

S_F = AGMA safety (reserve) **factor (often in the range of S_F=1.5~2)**

Y_N = stress cycle or life factor

Y_ϑ = temperature factor

Y_Z = reliability factor

Note: σ_{FP} is the allowable bending stress (at specific test conditions) for a given material whereas σ_{all} is the modified allowable bending stress with the consideration of factors such as life Y_N, temperature Y_ϑ and reliability Y_Z.

AGMA **allowable bending stress**

$$\sigma_{all} = \frac{\sigma_{FP}}{S_F} \frac{Y_N}{Y_\theta Y_Z}$$

σFP = **allowable bending stress** (MPa)

Tested at **10⁷** cycles and **99%** reliability, the **allowable bending stress** for **through hardened steels**

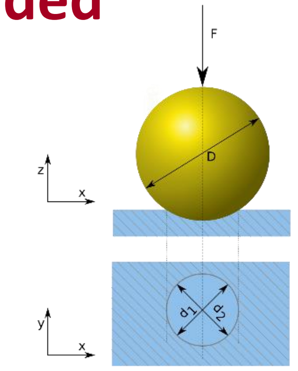
for **grade 1** steel gears: $\sigma_{FP} = 0.533 H_B + 88.3 (MPa)$

for **grade 2** steel gears: $\sigma_{FP} = 0.703 H_B + 113 (MPa)$

Tested at **10⁷** cycles and **99%** reliability, the **allowable bending stress** for **nitrided through hardened steels**

for **grade 1** steel gears: $\sigma_{FP} = 0.568 H_B + 83.8 (MPa)$

for **grade 2** steel gears: $\sigma_{FP} = 0.749 H_B + 110 (MPa)$



where, *HB* is the **Brinell hardness** (often in the range of ***HB*** = 160~400)

AGMA **allowable bending stress**

$$\sigma_{all} = \frac{\sigma_{FP}}{S_F} \frac{Y_N}{Y_\theta Y_Z}$$

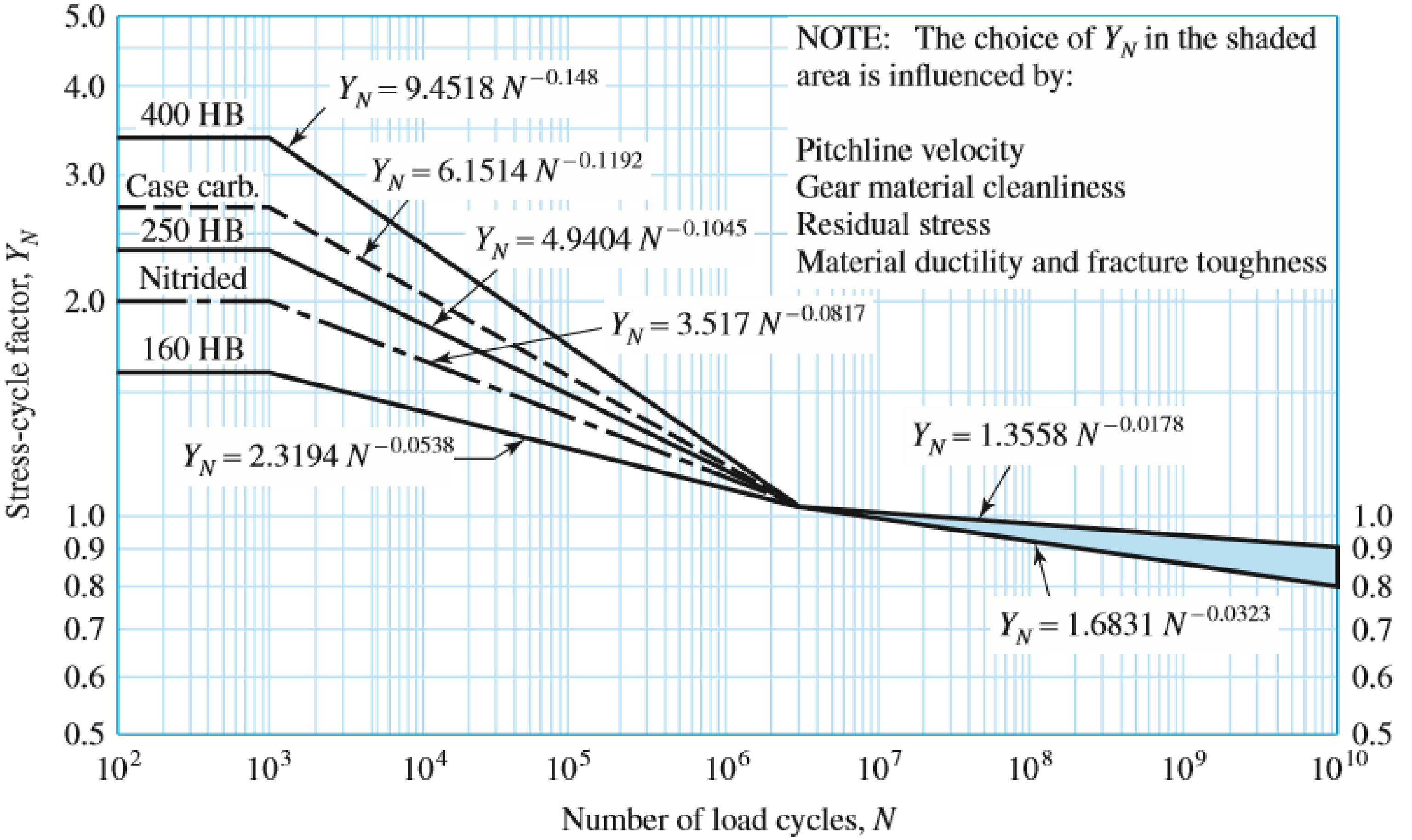
Reliability factor Table

Reliability	Y_Z
0.5	0.70
0.90	0.85
0.99	1.00
0.999	1.25
0.9999	1.50

Y_N = **life factor (stress-cycle factor)** for bending strength other than **10^7** cycles (also in **Appendix**).

Y_ϑ = **temperature factor**, for oil or gear temperature up to 120°C, ***Y_ϑ*** = 1.

Y_Z = **reliability factor**, to account for the statistical distribution of failure of material by fatigue.



AGMA **allowable contact stress** (ANSI/AGMA 2101-C95)

AGMA equation for the **allowable contact stress**

$$\sigma_{C,all} = \frac{\sigma_{HP}}{S_H} \frac{Z_N Z_W}{Y_\theta Y_Z}$$

σ_{HP} = allowable contact stress (MPa)

S_H = AGMA safety (reserve) **factor (often in the range of S_F=1.5~2)**

Z_N = stress cycle or life factor

Z_W = hardness ratio factor

Y_ϑ = temperature factor

Y_Z = reliability factor

Similarly, ***σ_{HP}*** is the **allowable contact stress** for a given material whereas ***$\sigma_{C,all}$*** is the **modified allowable contact stress** with the consideration of a number of factors as given above.

AGMA allowable contact stress

$$\sigma_{C,all} = \frac{\sigma_{HP}}{S_H}\frac{Z_N Z_W}{Y_\theta Y_Z}$$

σHP = **allowable contact stress** (MPa)

Tested at **10**^{7} cycles and **99%** reliability, the **allowable contact stress** for **through hardened steels**

for **grade 1** steel gears: $\sigma_{HP} = 2.22 H_B + 200 (MPa)$

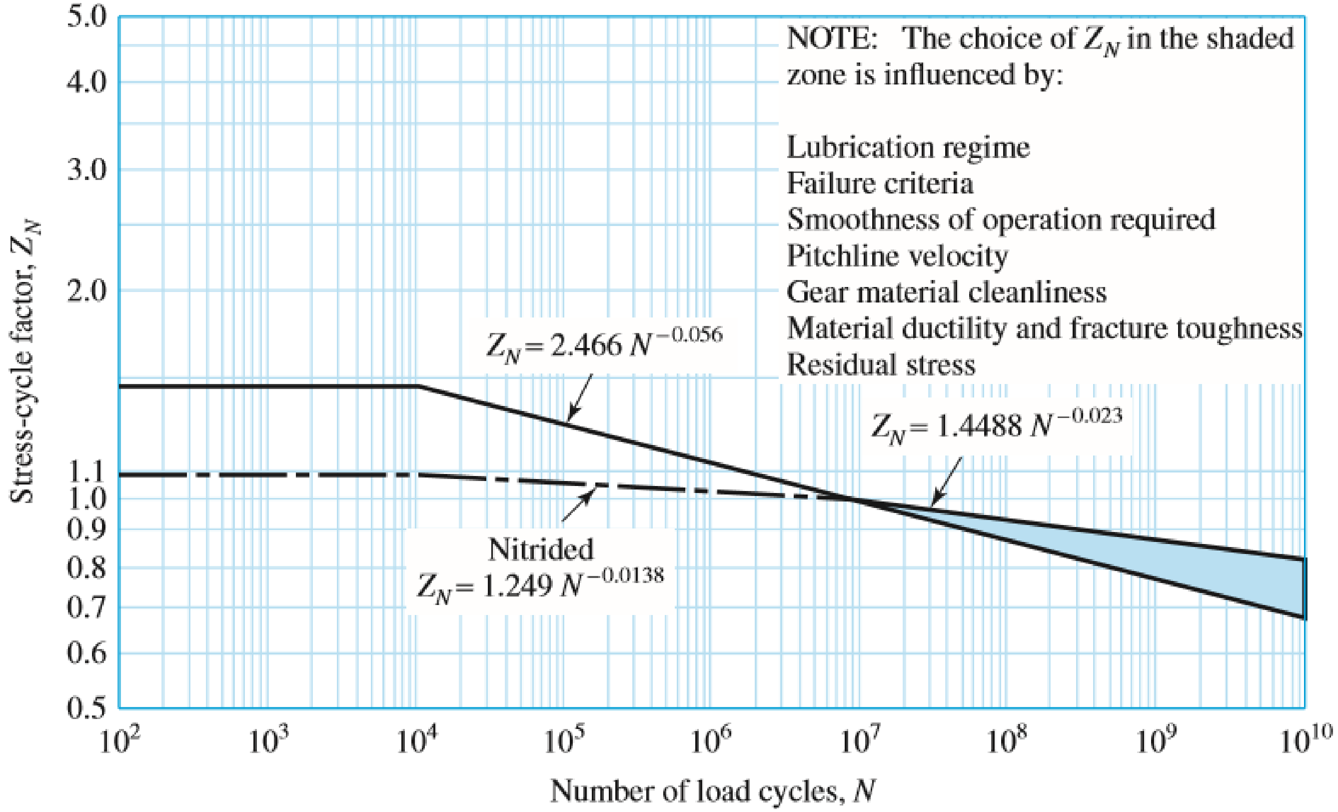
for **grade 2** steel gears: $\sigma_{HP} = 2.41 H_B + 237 (MPa)$

Note: between **250 HB and up to 450 HB** surface hardness can be obtained by through-hardening and Nitriding for **AISI 4340 and 4140 gear steels**.

AGMA **allowable contact stress**

$$\sigma_{C,all} = \frac{\sigma_{HP}}{S_H}\frac{Z_N Z_W}{Y_\theta Y_Z}$$

Z_N = **life factor (stress-cycle factor)** for pitting resistance other than **10^7** cycles (also in **Appendix**)



Z_W = **hardness ratio factor**, to account for different hardness of the pinion & gear (**only for gear, *i.e. Z_W = 1 for pinion***).

$$Z_W = 1 + A'(m_G - 1)$$

where,

$$A' = 8.98\times10^{-3}\left(\frac{H_{BP}}{H_{BG}}\right) - 8.29\times10^{-3}$$

for

$$1.2 \le \left(\frac{H_{BP}}{H_{BG}}\right) \le 1.7$$

where, HBP & HBG are the Brinell hardness of the **pinion** & **gear**

and,

$$m_G = \frac{N_G}{N_P}$$

Summary of AGMA **bending and contact stress analysis equations**

- **AGMA equations for bending & contact stresses**

Bending stress $$\sigma = W_t K_O K_V^{'} K_S \frac{1}{F\,m} \frac{K_H K_B}{Y_J}$$

Contact stress $$\sigma_C = Z_e \sqrt{W_t K_O K_V^{'} K_S \frac{K_H}{F d_p} \frac{Z_R}{Z_I}}$$

- **AGMA equations for allowable bending & contact stresses**

Allowable bending stress $$\sigma_{all} = \frac{\sigma_{FP}}{S_F} \frac{Y_N}{Y_\theta Y_Z}$$ ⇒ $$\sigma \leq \sigma_{all}$$

Allowable contact stress $$\sigma_{C,all} = \frac{\sigma_{HP}}{S_H} \frac{Z_N Z_W}{Y_\theta Y_Z}$$ ⇒ $$\sigma_C \leq \sigma_{C,all}$$

Key questions for gear design analysis

- What is the mechanism of forces transmitted and how to quantify them in gear meshing operations?

- What are the typical forms of stresses and how can they be accurately calculated?
  - ✓ Methods of determining stresses under ideal conditions
  - ✓ Considerations given to accommodate the effects of real working conditions **based on GAMA standard**

- With selected material and manufacturing methods, how to quantify the strength or allowable stress of a gear train?

A general procedure of gear design analysis

1) **Understand requirements** of function, power and speed, reliability, life, mounting & operation conditions.
2) **Select suitable gear types & trains**, e.g. simple and compound train, planetary train or differential unit (covered in the session Gears 2).
3) **Determine** numbers of teeth, gear ratio.
4) **Calculate** torque, **choose** a module, face width and **determine** pitch circle diameter.
5) **Select** a suitable material and hardness achieved by heat treatment.
6) **Calculate gear bending and contact stresses** using **AGMA** or **BS/ISO** standards.
   - ✓ Gear force analysis (transmitted load)
   - ✓ Calculation of gear **bending & contact stresses** & **allowable stresses**
7) If $\sigma \le \sigma_{all}$ and $\sigma_c \le \sigma_{c,all}$, acceptable; if not, go back to **4)** or **5)** and iterate

Normally a number of **iterations** of design & analysis are necessary to reach a satisfactory or optimum design solution.

Quiz: **True or False** to each of the following statements

Gears 3

A. **Scuffing** is a form of adhesive wear commonly caused by lack of lubrication in high speed gear system.

B. **Transmitted load** is the radial component of the exerted force in a pair of spur gears.

C. **Gear bending stress** calculation is based on Hertzian contact theory.

D. **Contact stress** is localised stresses created between two curved bodies under loading.

E. In gear design analysis, **AGMA standard** is commonly used to calculate gear **bending and contact stresses** in operation conditions.

F. **AGMA allowable bending and contact stresses** are dependent upon chosen material, treatment and other operational conditions.

Quiz: **True or False** to each of the following statements

Gears 3

A. **Scuffing is a form of adhesive wear commonly caused by lack of lubrication in high speed gear system.** **(true)**

B. **Transmitted load is the radial component of the exerted force in a pair of spur gears.** **(false)**

C. **Gear bending stress calculation is based on Hertzian contact theory.** **(false)**

D. **Contact stress is localised stresses created between two curved bodies under loading.** **(true)**

E. **In gear design analysis, AGMA standard is commonly used to calculate gear bending and contact stresses.** **(true)**

F. **AGMA allowable bending and contact stresses are only dependent upon chosen material.** **(false)**

Revision questions

- What are the common forms of gear failure and what are the root causes for these failures?
- Can you calculate the transmitted load of a gear system with given power and rotating speed?
- Can you explain the general methods used to calculate gear bending and contact stresses?
- What is the main difference between the basic Lewis bending and Hertz contact stress equations and those used in AGMA standard?
- Why are so many factors, e.g. life factors, Y_N or Z_N, used in calculating the AGMA allowable bending and contact stresses?
- In gear design, what would you do if the calculated bending or contact stress is larger than the allowable bending or contact stress?

Gear Design Resources

- **Budynas, R.C., Nisbett, J.K., 2015, Shigley's Mechanical Engineering Design, 10th edition, McGraw-Hill** (TJ230 SHI)
  - **Chapter 14 Spur and Helical Gears**
- **Shigley, J.E., Mischke, C.R., Budynas, R.G., 2003. Mechanical Engineering Design, 7th edition, McGraw-Hill** (TJ230 SHI)
  - **Chapter 14 Spur and Helical Gears**
- **Childs, R.N., 2014. Mechanical Design Engineering Handbook, Elsevier,** (TJ230 CHI)
  - **Chapter 9 Spur and Helical Gear Stressing (*online version available via NUSearch*)**
- ANSI/AGMA 2101-C95, Fundamental rating factors and calculation methods for involute spur gear and helical gear teeth, (Metric edition of ANSI/AGMA 2001-C95)
- BS ISO 6336-1~6: 2006, British and ISO Standards of gear load capacity, calculation of tooth bending strength & surface durability, strength and quality of materials (***available via BSI website***).

Gears 3

End of Part 3

Gears 3

Part 4

Worked example 2: A case study of gear stress analysis and design*

*** This case study is an extract from** Childs' Book on ***Mechanical Design Engineering Handbook***, 2014, Elsevier, Ch9, pp386-391 (available online via NUSearch)

- **Calculate AGMA bending and contact stresses and the factors of safety (reserve factor)** for both the pinion and gear of a simple spur gear drive.

Design specifications:

- **Numbers of teeth** for both pinion and gear are N_P**=18,** N_G**=50** with a **pressure angle ϕ=20°.**
- The **module** is ***m*=2.5 mm** and the **face width** is ***F*=30 mm**.
- The **rotating speed** of the pinion is n_P**=1425 rpm** and the gear drive is to transmit a **power** of ***P*=3 kW** under smooth running conditions.
- The gear material is **grade 1 steel** with the **Brinell hardness** to be H_B**=240** and **200** for the pinion and gear, respectively.
- The gears are manufactured to Q_v**=6** AGMA accuracy and the teeth are **uncrowned**.
- The gear drive is required to have a **life** of **10^8 cycles** and **90% reliability.**
- The gears are straddle mounted within an enclosed unit.

Worked example 2: A case study of gear stress analysis and design*

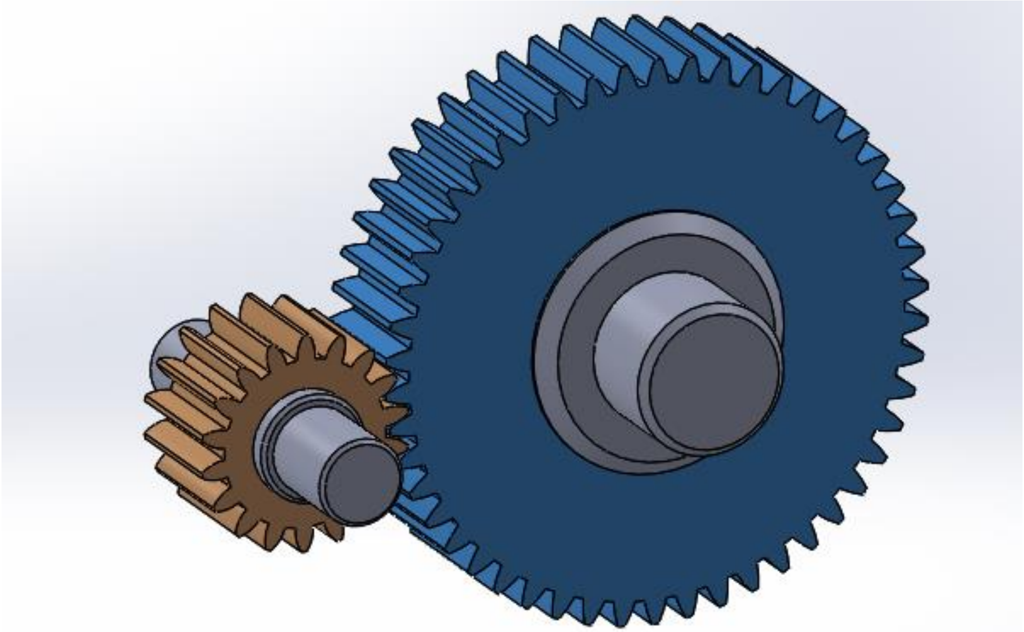
*** This case study is an extract from** Childs' Book on ***Mechanical Design Engineering Handbook***, 2014, Elsevier, Ch9, pp386-391 (available online via NUSearch)

Design specifications:

- **Numbers of teeth: N_P=18, N_G=50; pressure angle: ϕ=20°**
- Module: ***m*=2.5 mm; face width: *F*=30 mm**
- **Rotating speed of the pinion: n_P=1425 rpm; power: *P*=3 kW**

See separate Handouts available on Moodle for detailed calculations.

Be aware there is a corrected error in gear contact stress result.



Solidworks assembly model without details of shafts, keys/keyways & bearings, etc

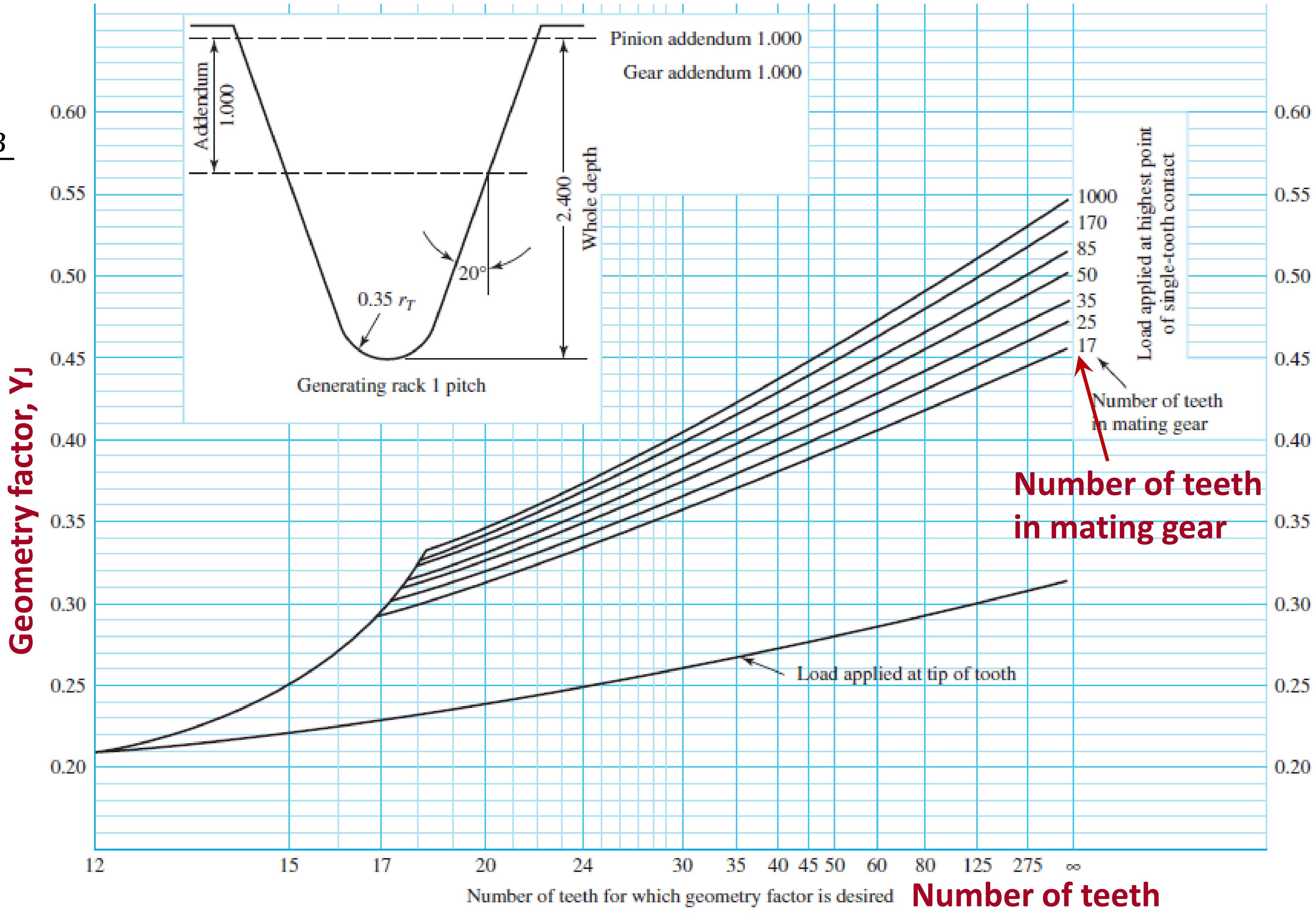
Gears 3

End of session

Appendix: AGMA charts of gear bending and contact stress factors

$$\sigma = W_t K_O K_V^{'} K_S \frac{1}{Fm} \frac{K_H K_B}{Y_J}$$

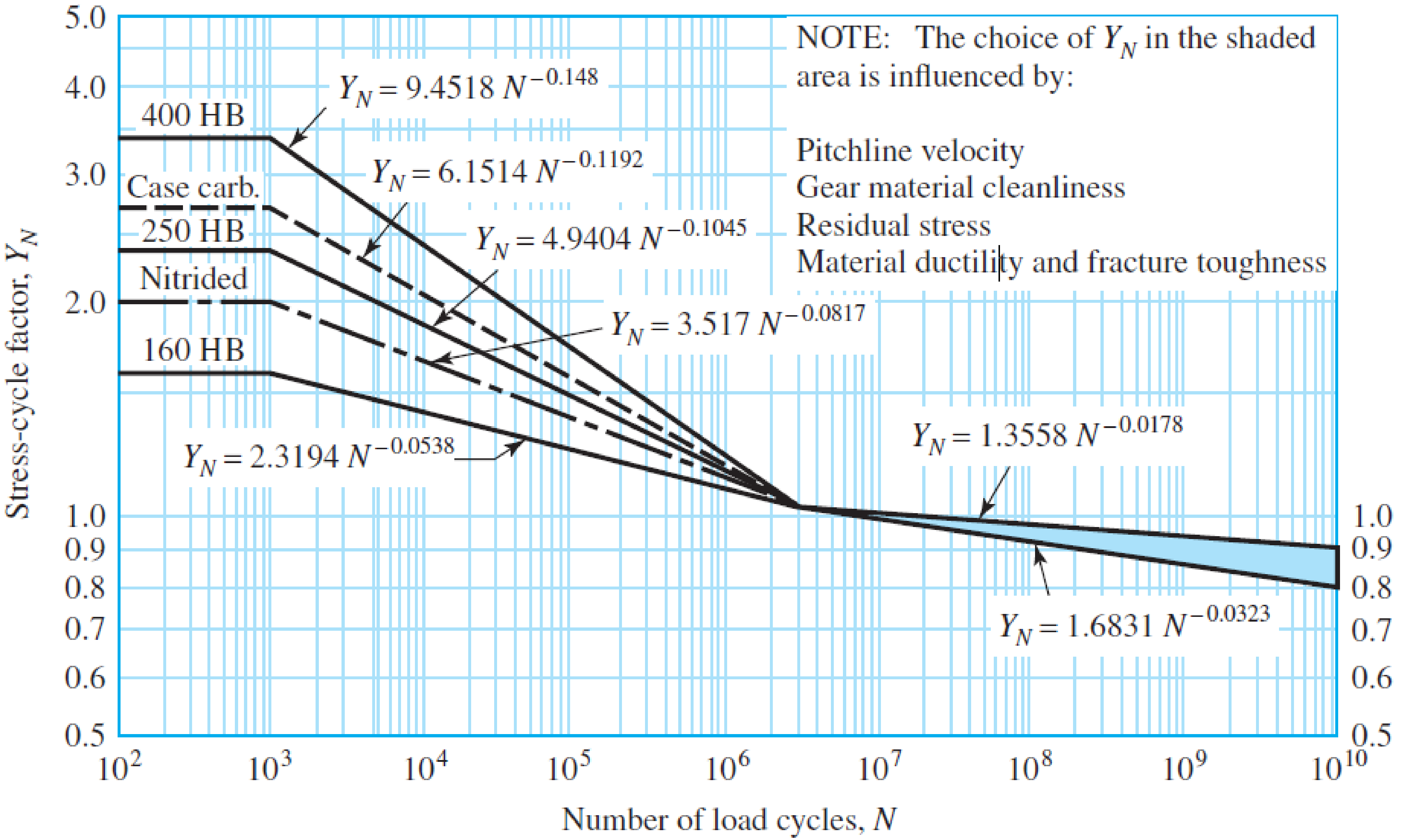
where Y_J = **geometry factor** for bending strength



Appendix: AGMA Charts of gear bending and contact stress factors

$$\sigma_{all} = \frac{\sigma_{FP}}{S_F}\frac{Y_N}{Y_\theta Y_Z}$$

where Y_N = **life factor (or stress cycle factor)** for **allowable bending stress**



Appendix: AGMA Charts of gear bending and contact stress factors

$$\sigma_{C,all} = \frac{\sigma_{HP}}{S_H}\frac{Z_N Z_W}{Y_\theta Y_Z}$$

where Z_N = **life factor (or stress cycle factor)** for **allowable contact stress**